

IDENTIFICATION AND COMPENSATION OF WIENER-HAMMERSTEIN SYSTEMS WITH FEEDBACK

Andrew Bolstad, Benjamin A. Miller, Joel Goodman, James Vian and Janani Kalyanam

MIT Lincoln Laboratory
Lexington MA 02420

ABSTRACT

Efficient operation of RF power amplifiers requires compensation strategies to mitigate nonlinear behavior. As bandwidth increases, memory effects become more pronounced, and Volterra series based compensation becomes onerous due to the exponential growth in the number of necessary coefficients. Behavioral models such as Wiener-Hammerstein systems with a parallel feedforward or feedback filter are more tractable but more difficult to identify. In this paper, we extend a Wiener-Hammerstein identification method to such systems showing that identification is possible (up to inherent model ambiguities) from single- and two-tone measurements. We also derive the Cramér-Rao Bound for the system parameters and compare to our identification method in simulation. Finally we demonstrate equalization performance using measured data from a wideband GaN power amplifier.

Index Terms— Nonlinear system identification, Wiener-Hammerstein system, nonlinear equalization, predistortion, log-frequency

1. INTRODUCTION

Operating RF power amplifiers in compression can produce unwanted nonlinear distortions in transmitted signals causing both in-band distortions and adjacent channel interference. Such distortions must be compensated to maintain high power efficiency. For narrowband systems, look up tables (LUTs) provide sufficient digital predistortion; however, wideband amplifiers exhibit memory effects which require more sophisticated techniques and system models. Models based on (often “pruned”) Volterra series can model almost any nonlinear system, but their complexity increases quickly (exponentially in some cases) with memory and model order, making them difficult to implement [1, 2, 3, 4, 5].

Alternatively, behavioral models such as Wiener-Hammerstein (WH) offer more parsimonious representations

of many nonlinear systems [1]. These models contain a memoryless nonlinearity cascaded between two linear FIR filters and thus are capable of representing nonlinearities with short memory effects. WH systems can be identified by examining responses to a series of single- and two-tone excitations in the log-frequency domain [6]. A richer class of systems can be modeled with the addition of a feedback path to the WH system (Fig. 1(a)). These Wiener-Hammerstein with Feedback (WHFB) system models maintain the parsimonious nature of WH systems while allowing for longer term memory effects; however, the added flexibility of a feedback path in the WHFB model complicates system identification.

In this paper, we extend the log-frequency approach of [6] to the identification of WHFB systems. We exploit the fact that in many cases the WHFB system can be inverted with a Wiener-Hammerstein with Feedforward (WHFF) system. We analyze the inherent ambiguities of the system and derive the Cramér-Rao Bound (CRB) for the system parameters. The performance of the approach is demonstrated in both simulations and on a GaN high power amplifier (HPA).

The remainder of this paper is organized as follows. We briefly survey previous work in Sec. 2. In Section 3 we describe the WHFB and WHFF system models. The log-frequency identification technique is developed in Section 4. Ambiguities and the CRB are discussed in Sec. 5, while simulations and measured data are presented in Sec. 6. We conclude with a summary.

2. PREVIOUS WORK

Nonlinear system modeling and compensation has been the subject of significant research (see e.g. [1, 7]). The WHFB system in particular appears in [8], but its identification is not discussed. Silva et al. describe a method for identifying a modified WHFF system model of a traveling wave tube amplifier [9], but assume the response of the input filter can be measured in isolation. Vandersteen and Schoukens study a model similar to the WHFB system in [10]. Our method is similar to their magnitude response initialization step, but their phase response estimate relies on an iterative procedure. The identification of a WHFF system as the inverse to a WHFB system is unique to our approach.

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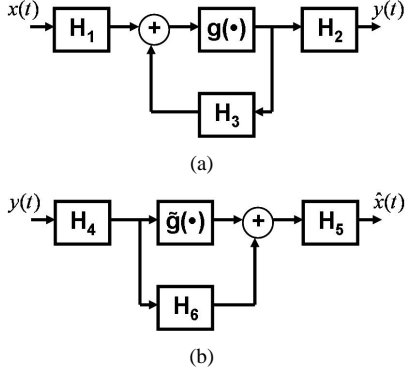


Fig. 1. Two nonlinear behavioral models: (a) Wiener-Hammerstein with Feedback and (b) Wiener-Hammerstein with Feedforward. These systems can invert each other.

3. THE WIENER-HAMMERSTEIN WITH FEEDBACK SYSTEM

The WHFB system shown in Fig. 1(a) consists of an input filter $H_1(\omega)$, an output filter $H_2(\omega)$, a memoryless, invertible nonlinearity $g(x)$, and a filter $H_3(\omega)$ in a feedback loop. Under mild assumptions, the Wiener-Hammerstein with *feedforward* system of Fig. 1(b) inverts the WHFB system when $H_4 = H_2^{-1}$, $H_5 = H_1^{-1}$, $H_6 = -H_3$, and $\tilde{g} = g^{-1}$.

The memoryless nonlinearity $\tilde{g}(x)$ can be approximated by a polynomial of sufficiently high degree. We assume a strictly third order polynomial $\tilde{g}(x) = \gamma_3 x|x|^2$, where we incorporate linear term $\gamma_1 x$ into H_6 . Incorporating higher order terms into the estimation procedure is straightforward; however, better results are often obtained by using estimated filter responses from the third order model to create an LUT representation of \tilde{g} .

4. LOG-FREQUENCY IDENTIFICATION

To describe the identification procedure it is convenient to cascade the WHFB and WHFF models such that the output of the device under test, modeled by the feedback system, is fed into an inverter, though in practice the WHFF system can be used as a nonlinear digital predistorter. The WHFB system is probed with single- and dual-tone training signals denoted $x(t)$. Treating $y(t)$ as input and $x(t)$ as desired output, we build a system of equations to identify the WHFF system.

For single-tone training signals, the power is scaled back so that a linear approximation of the system is sufficient. The output of the WHFB system is a single tone, and the desired filters H_4 , H_5 , and H_6 obey:

$$X(\omega_1) \approx H_4(\omega_1)H_5(\omega_1)H_6(\omega_1)Y(\omega_1). \quad (1)$$

For the two-tone training signals at frequencies ω_1 and ω_2 , the power is increased so that third order intermodulation

products (intermods) present in y are above the noise floor, but higher order distortions are still negligible. This means $Y(\omega)$ has nonnegligible components at frequencies ω_1 , ω_2 , $2\omega_1 - \omega_2$, and $2\omega_2 - \omega_1$. The intermods are typically weak enough to experience only linear effects of the WHFF system, while the components $Y(\omega_1)$ and $Y(\omega_2)$ produce additional intermods as they pass through the WHFF system. Since the desired signal $X(\omega)$ has no energy at the intermod frequencies, the new intermods created by the WHFF system must cancel the linear responses to $Y(2\omega_1 - \omega_2)$ and $Y(2\omega_2 - \omega_1)$. This leads to the following constraint at frequency $2\omega_1 - \omega_2$ (an analogous constraint exists at frequency $2\omega_2 - \omega_1$):

$$\begin{aligned} H_4(2\omega_1 - \omega_2)H_5(2\omega_1 - \omega_2)H_6(2\omega_1 - \omega_2)Y(2\omega_1 - \omega_2) \\ = 3\gamma_3 H_4^2(\omega_1)H_4^*(\omega_2)H_5(2\omega_1 - \omega_2)Y^2(\omega_1)Y^*(\omega_2). \end{aligned} \quad (2)$$

Taking logarithms of Eqs. (1) and (2) and collecting known terms into variables $Y_{a,i}$ and $Y_{p,i}$ allows us to write linear equations for both the log-magnitudes and phases of the unknown filter responses and third order coefficient γ_3 (assumed positive here):

$$Y_{a,l}(\omega_1) = \alpha_4(\omega_1) + \alpha_5(\omega_1) + \alpha_6(\omega_1) \quad (3)$$

$$\begin{aligned} Y_{a,nl}(\omega_1, \omega_2) = 2\alpha_4(\omega_1) + \alpha_4(\omega_2) - \alpha_4(2\omega_1 - \omega_2) \\ - \alpha_6(2\omega_1 - \omega_2) + \log |\gamma_3| \end{aligned} \quad (4)$$

$$Y_{p,l}(\omega_1) = \theta_4(\omega_1) + \theta_5(\omega_1) + \theta_6(\omega_1) \quad (5)$$

$$\begin{aligned} Y_{p,nl}(\omega_1, \omega_2) = 2\theta_4(\omega_1) - \theta_4(\omega_2) - \theta_4(2\omega_1 - \omega_2) \\ - \theta_6(2\omega_1 - \omega_2) \end{aligned} \quad (6)$$

where $\alpha_i(\omega) = \log |H_i(\omega)|$ and $\theta_i(\omega) = \angle H_i(\omega)$.

Evaluating the WHFB system at $n = 1, 2, \dots, N$ uniformly spaced frequencies over bandwidth B results in $3N+1$ unknown log-magnitude terms and $3N$ unknown phase terms. Training with single-tones at each frequency provides N linear equations for both log-magnitude and phase. Two-tone signals provide additional equations, but achieving full rank systems of equations is impossible due to inherent ambiguities in the WHFF and WHFB systems as we show in Sec. 5.1. We use a weighted least squares approach to find a solution.

Phase wrapping also complicates the identification of the filter phase responses since all phase equations hold modulo 2π . To handle this problem, P undergoes a two-dimensional phase-unwrapping based on [11, 12].

5. IDENTIFICATION PERFORMANCE LIMITS

5.1. Model Ambiguities

The WHFF model cannot be determined uniquely from training signals and output data, no matter what model identification technique is used. Inherent ambiguities allow two WHFF systems with different parameters to appear identical from

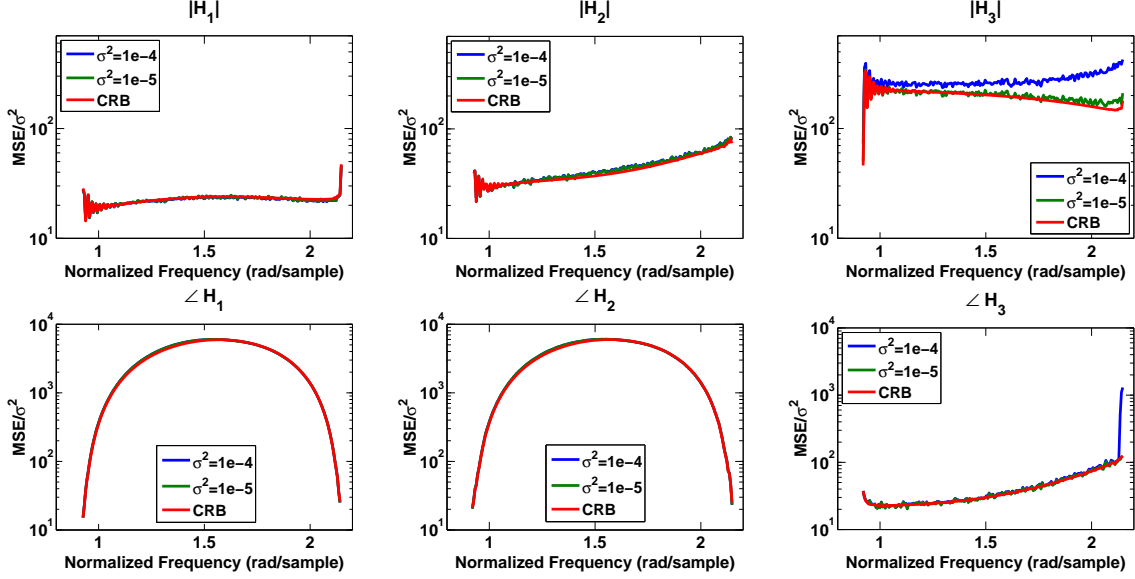


Fig. 2. Performance when identifying a WHFF model. Normalized variance in amplitude (top row) and phase (bottom row) estimation are compared to the CRB for H_4 , H_5 and H_6 (left, center and right columns, respectively).

an input/output perspective. Understanding these ambiguities aids in the design of nonlinear digital compensation systems.

The system of equations described by Eqs. (3 – 6) evaluated over a set of training signals can be written as two matrix equations, one in log-magnitude, one in phase:

$$\begin{bmatrix} \mathbf{y}_{a,l} \\ \mathbf{y}_{a,nl} \end{bmatrix} = \begin{bmatrix} \mathbf{M}_{l,4} & \mathbf{M}_{l,5} & \mathbf{M}_{l,6} & 0 \\ \mathbf{M}_{nl,4} & \mathbf{M}_{nl,5} & \mathbf{M}_{nl,6} & 1 \end{bmatrix} \begin{bmatrix} \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \log |\gamma_3| \end{bmatrix} \quad (7)$$

$$\begin{bmatrix} \mathbf{y}_{p,l} \\ \mathbf{y}_{p,nl} \end{bmatrix} = \begin{bmatrix} \mathbf{P}_{l,4} & \mathbf{P}_{l,5} & \mathbf{P}_{l,6} \\ \mathbf{P}_{nl,4} & \mathbf{P}_{nl,5} & \mathbf{P}_{nl,6} \end{bmatrix} \begin{bmatrix} \theta_4 \\ \theta_5 \\ \theta_6 \end{bmatrix}. \quad (8)$$

Examination of the coefficients in Eqs. (3) and (5) reveals that a small signal sweep over frequencies $n = 1, 2, \dots, N$ leads to $\mathbf{M}_{l,4} = \mathbf{M}_{l,5} = \mathbf{M}_{l,6} = \mathbf{P}_{l,4} = \mathbf{P}_{l,5} = \mathbf{P}_{l,6} = \mathbf{I}_N$. Similarly, Eqs. (4) and (6) describe a row of matrices $\mathbf{M}_{nl,i}$ and $\mathbf{P}_{nl,i}$ when tones at frequencies n_1 and n_2 are used as training signals. In particular, $\mathbf{M}_{nl,5} = \mathbf{P}_{nl,5} = \mathbf{0}$, while each row of $\mathbf{M}_{nl,4}$ (resp. $\mathbf{P}_{nl,4}$) will have a 2, 1 (resp. -1), and -1 in columns n_1 , n_2 , and $2n_1 - n_2$. The corresponding row of matrices $\mathbf{M}_{nl,6}$ and $\mathbf{P}_{nl,6}$ will have a single -1 in column $2n_1 - n_2$. Given the structure of each row of these matrices, the following can be verified:

Theorem 1: The nullspace in Eq. (7) contains the vectors:

$$\begin{aligned} \mathbf{v}_1 &= [\mathbf{1}_N^T \quad -3\mathbf{1}_N^T \quad 2\mathbf{1}_N^T \quad 0]^T \\ \mathbf{v}_2 &= [\mathbf{1}_N^T \quad -\mathbf{1}_N^T \quad \mathbf{0}_N^T \quad -2]^T \end{aligned}$$

while the nullspace in Eq. (8) contains the vectors:

$$\begin{aligned} \mathbf{w}_1 &= [-1 \quad -2 \quad \dots \quad -N \quad 1 \quad 2 \quad \dots \quad N \quad \mathbf{0}_N^T]^T \\ \mathbf{w}_2 &= [\mathbf{1}_N^T \quad -\mathbf{1}_N^T \quad \mathbf{0}_N^T]^T. \end{aligned}$$

These nullspace vectors are not a deficiency in the identification procedure; instead they reveal ambiguities of the model itself. Vector \mathbf{v}_1 indicates that multiplying the gain of H_4 at all frequencies by a constant c_1 can be counteracted by multiplying the gain of H_5 by c_1^{-3} and H_6 by c_1^2 . Similarly, \mathbf{v}_2 shows that scaling H_4 by c_2 is offset by scaling H_5 by c_2^{-1} and γ_3 by c_2^{-2} . As for the phase, \mathbf{w}_1 indicates that adding a delay of τ_1 to H_4 can be offset by adding an advance of τ_1 to H_5 . Vector \mathbf{w}_2 corresponds to applying a constant phase shift ψ at all (positive) frequencies (similar to a Hilbert transform) and a corresponding phase shift by $-\psi$ at all (positive) frequencies in filter H_6 .

5.2. Cramér-Rao Bound

The Cramér-Rao Bound (CRB) provides a lower bound on the variance an unbiased estimator can achieve independent of the estimation technique [13]. Due to the inherent ambiguities in the WHFF model, we fix the magnitude gains for the first frequency bin of H_4 and H_5 and the phase response for the first and last frequency bin of H_4 so that the CRB for the remaining parameters can be found.

We simulate the WHFF system response to 1575 one- and two-tone test signals and add white Gaussian noise at about -15 and -25 dB (relative to intermod power). We compare the mean squared error (MSE) averaged over 1000 trials to the CRB in Fig. 2 where the MSE has been normalized by

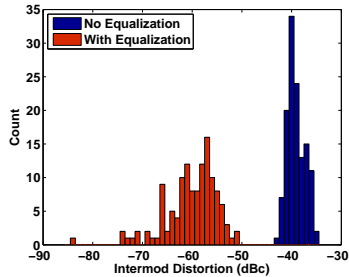


Fig. 3. Reduction in power of third order intermods when the WHFF inverter is applied to validation data. Average reduction is about 20dBc.

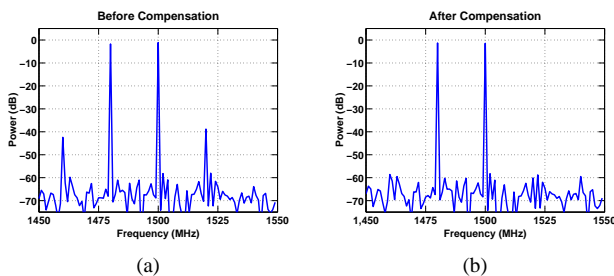


Fig. 4. Measured performance of the WHFF inverter applied to a two-tone verification signal. Before inversion (a) the intermods are 20 dB larger than after inversion (b).

the noise power. The performance of the log-magnitude estimation technique matches the CRB for nearly all parameters with some deviation in the estimate of $|H_6|$ at lower SNR, particularly at higher frequencies.

6. MEASURED RESULTS

Modeling a wideband HPA as a WHFB system, we applied the log-magnitude identification technique to estimate parameters of a WHFF inverter. The band of interest spanned 100 MHz (1450–1550 MHz), and we estimated the filter responses at 1 MHz increments. In addition to sweeping a single tone, small amplitude test signal across the band, we collected two-tone data with spacings of 1, 2, 5, 10, 20, and 40 MHz and used half for model identification and half for verification. Data were collected using the Combat Adaptive Radio Testbed (CART) [14].

The identified WHFF inverter was applied to verification data as a nonlinear digital equalizer (i.e. post-inverse), though the same structure could be used for nonlinear predistortion as well. Figure 3 shows the performance of the identified WHFF inverter. Averaging over all 127 verification data sets yields an average distortion reduction of 20 dBc. A typical example is shown in Fig. 4.

7. SUMMARY

We have presented a novel identification procedure for Wiener-Hammerstein with Feedforward systems, as well as Wiener-Hammerstein with Feedback systems through the inverse relationship described in Sec. 3. We have identified inherent system ambiguities which have no effect on the input/output relationship of the system, and derived the Cramér-Rao Bound for estimation of system parameters. Simulations show the technique often attains the CRB, and measured results demonstrate the effectiveness of both the model and identification procedure on a high power amplifier.

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