Monetary Policy Shocks and Local Housing Prices: 
the Transmission Mechanism *

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Abstract

This paper examines the differential effect of monetary policy shocks on different U.S. local housing markets. Conventional methods focus solely on aggregate housing prices, but neglect useful cross-sectional variation in local housing prices. By exploiting the heterogeneity in housing supply elasticity, I provide estimates of local housing price responses to monetary policy shocks in a large sample of metropolitan statistical areas. Given an expansionary shock that decreases the Federal Funds rate by 100 basis points, housing prices increase by 12% in cities with a highly inelastic housing supply (e.g., San Francisco), but by only 1.7% in cities with a very elastic housing supply (e.g., Iowa City) at the two-year horizon. To understand the monetary policy transmission mechanism in the housing market, I develop a structural model of the housing price with information friction. The model is identified by incorporating the variation in housing supply and estimated by targeting model-implied housing price impulse responses to their empirical counterparts. Structural estimates of the model suggest that households are well-informed about changes in local housing demand but have little idea about how changes in monetary policy affect the local housing market.

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1 Introduction

Monetary policy plays an important role in driving housing market dynamics. The large price boom in the U.S. housing market in 2000s stirred up discussions on the effect of monetary policy on housing prices. While Taylor (2007) finds that excessively low policy rates fueled the housing bubble, Bernanke (2010) defended the Fed by arguing that links between the housing boom and monetary policy are weak. Local housing markets exhibited large degrees of heterogeneity during the boom. Conventional studies on the effect of monetary policy focus solely on aggregate housing prices, but neglect useful cross-sectional variations in local housing prices.

This paper estimates the differential effect of aggregate monetary policy shocks on U.S. local housing markets. Monetary policy shocks are identified through high-frequency identification methods. By exploiting the heterogeneity in housing supply elasticities, I provide estimates of dynamic local housing price responses to monetary policy shocks in a large sample of metropolitan statistical areas (MSA). Given an expansionary shock that decreases Federal Funds rate (FFR) by 100 basis points, housing prices increases by 12% in cities with highly inelastic housing supply (e.g. San Francisco) but by only 1.7% in cities with very elastic housing supply (e.g. Iowa City) at the two year horizon.

The estimated IRF suggests that housing prices respond very sluggishly to monetary policy shocks. To understand the housing market’s monetary policy transmission mechanism, I develop a structural model of housing prices with a focus on information frictions. I provide estimates on the size of information frictions that align the adjustment speed of housing prices in the model with my empirical findings. I identify model parameters by incorporating the cross-sectional variation in housing supply and estimate them by targeting model-implied housing price impulse responses to their empirical counterparts. Estimates of the model suggest that households are well informed about changes in local housing demand but have little idea about how changes in monetary policy affect the local housing market.

I use externally identified monetary policy shocks following the high-frequency identification literature (see Kuttner [2001], Barakchian and Crowe [2013], Swanson [2017]). I apply the identified monetary policy shocks in two different frameworks. First, I extend the Jordà [2005] local projection
method to a panel setting. By interacting monetary policy shocks with housing supply elasticities, I exploit both cross-sectional and time-series variations in the housing price growth. This specification imposes an inverse linear relationship between housing supply elasticity and the size of the housing price response. I find near-zero responses in housing prices for the first year but large and persistent responses from the second year onward. The second method is based on a panel proxy VAR model employing high-frequency-identified shocks as an instrument for monetary policy shocks. In this specification, I do not impose the cross-sectional identification restriction. Instead, I group MSA-level housing prices to four quartiles based on their housing supply elasticities. I find that the results roughly align with the prediction that housing prices of cities in more inelastic quartiles increase more after an expansionary monetary policy shock. The response at the two-year horizon is about three quarters as large as the estimate from Jordà local projection.

It is hard to reconcile the housing price response's rigidity to monetary policy shocks with predictions from rational expectation models. If households know that an expansionary monetary policy shock would drive up future housing prices, why did they wait for a year or longer to act upon this good news? To solve this puzzle, I propose a model of housing prices with information frictions. The model is based on Glaeser and Nathanson (2017), which I generalize to allow for endogenous housing supplies and monetary policy shocks. The households' information sets include local housing prices, noisy signals about the local demand and noisy monetary policy signals. I provide analytical solutions to the household's signal-extraction problem to derive equilibrium housing price dynamics. The size of noise in the households' signals measures the degree of information frictions in the transmission of monetary policy shocks.

I present a simple method to estimate this structural model. I target the model-predicted housing price IRFs to their empirical counterparts. This method provides a general framework to estimate structural parameters in models with aggregate shocks and heterogeneous cross-sectional impacts. The estimates suggest that households pay little attention to changes in the monetary policy stance. The results show that, in a rational expectation framework, explaining the sluggishness of housing prices in response to monetary policy shocks requires a significant size of information friction.

Another possible explanation of rigidity is to use search models for housing. Head et al. (2014)
build a housing search model with housing construction and calibrate it to the IRFs generated from a panel VAR. They show that searching improves the fit of housing price IRFs to income shocks, but its impact effect is about 80% of the overall response. Guren (2018) develop a search model to explain the housing price momentum by adding strategic complementarity and buyers with extrapolative beliefs. As an extension to my model, I show a simple method to allow for search frictions. However, adding search frictions only mildly reduces the response in the first six months.

I apply my estimates on housing price responses to explore monetary policy’s impact on the housing boom in the 2000s. In particular, I provide an assessment of Taylor’s criticism of the Fed’s loose monetary policy for driving the housing boom. I first estimate the path of interest rates predicted by the Taylor rule and its deviation from the FFR. To evaluate the contribution of this deviation to the housing boom, I accumulate its effect on housing prices using my estimated IRFs. I present results for San Francisco and Oklahoma City, and show that deviations from the Taylor rule explains only about a quarter of the housing boom.

The plan for the paper is as follows. Section 2 presents the identification of monetary policy shocks and the empirical estimation of housing price IRFs. Section 3 proposes and solves a housing price model with information frictions. The structural estimation of this model is presented in Section 4. Section 5 discusses the application of my results to the 2000s housing boom. Section 6 concludes.

2 Empirical evidence

This section presents the empirical evidence on the heterogeneous effects of aggregate monetary policy shocks on local housing prices. Monetary policy shocks are identified through a high-frequency identification method. Section 2.1 discusses how I identify and estimate the shock, as well as its relationship to other identification methods in the literature. I explain the cross-sectional variation in housing prices using local housing supply conditions. Section 2.2 presents data on the housing price index and housing supply elasticities. I estimate the IRF of local housing prices to monetary policy shocks in two different ways. Section 2.3 discusses the Jordà local projection method, where I directly impose an inverse relationship between housing supply elasticities and the size of housing
price responses through the panel structure. Section 2.4 presents the panel proxy SVAR method using the identified shocks as an instrument for actual monetary policy shocks. I discuss the pros and cons of each method and show that they provide quantitatively similar estimates.

2.1 Identification of monetary policy shocks

I identify shocks to monetary policy following the high-frequency-identification (HFI) literature. To make an inference about the unexpected component of monetary policy announcements, I study movements in financial markets within a narrow window around Federal Open Market Committee (FOMC) press releases. There are about eight FOMC press releases a year, each following a regularly scheduled meeting and some occasional intermeeting announcements.

To measure the unanticipated component of each FOMC press release, I study several financial instruments of varying maturities including federal funds futures, Eurodollar futures, and Treasury bills. I compute the first principal components of these asset price responses in the 30-minute windows around the FOMC announcement between January 1991 and June 2008. Let a matrix $X$ summarize changes in asset prices with rows corresponding to FOMC announcements and columns corresponding to $n$ different assets; each element $x_{ij}$ of $X$ then reports the 30-minute response of the $j$th asset to the $i$th FOMC announcement. As in Gürkaynak et al. (2005), we can think of these data in terms of a factor model, $X = F\Lambda + \epsilon$ where $F$ is a matrix of unobserved factors, $\Lambda$ is a matrix of factor loadings, and $\epsilon$ is a matrix of white noise residuals. Following Swanson (2017), asset prices include the first and third federal funds futures contracts, the second, third, and fourth Eurodollar futures contracts, and the 2-, 5-, and 10-year Treasury yields. Each column of $F$ can be estimated with principal-component analysis. Since the scale of $F$ and $\Lambda$ are indeterminate, I normalize each component of $F$ such that a one-unit increase in the factor leads to a 100-basis-point increase in 10-year Treasury Bill. Loadings on the first factor are reported in Table 1.

This identification of monetary policy shocks involves more financial assets than most earlier literature using high-frequency identifications. Gorodnichenko and Weber (2016), Nakamura and

\footnote{As is standard in the literature, these contracts are scaled by the number of days remaining in the month to provide an effective estimate of the surprise change in the FFR after the announcement (see Gürkaynak et al., 2005; Kuttner, 2001 for details).}
Table 1: Loadings on changes in asset prices on FOMC meeting days

<table>
<thead>
<tr>
<th></th>
<th>MP1</th>
<th>MP2</th>
<th>ED2</th>
<th>ED3</th>
<th>ED4</th>
<th>2y Tr.</th>
<th>5y Tr.</th>
<th>10y Tr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loadings</td>
<td>0.2537</td>
<td>0.3069</td>
<td>0.3951</td>
<td>0.4012</td>
<td>0.3895</td>
<td>0.4009</td>
<td>0.3584</td>
<td>0.2888</td>
</tr>
</tbody>
</table>

Notes: This table reports the loading on the first factor. The assets include the first (MP1) and third (MP2) federal funds futures contracts, the second (ED2), third (ED3), and fourth (ED4) Eurodollar futures contracts, and the 2-, 5-, and 10-year Treasury yields. The sum-of-squares of the loadings equals 1.

Steinsson (2018) and others, consider changes only in the current month federal funds futures contract. Barakchian and Crowe (2013) uses the first factor of changes in six federal funds futures contracts up to five months. As argued in Swanson (2017), monetary policy communications have more than one dimension. By constructing the shock based on asset prices covering longer maturities, this measure captures the persistent component of monetary policy shocks, which is the most relevant for the study of housing markets. But when using other high-frequency identification shocks, my results are similar to those presented below. The common key identifying assumption among this and other studies is that there are no other factors that occurred within the window around the FOMC announcement that moved the financial instruments.

To construct monthly measures of monetary policy shocks, I aggregate the high-frequency estimates by simple summation within any month $t$. If there is no FOMC meeting in a particular month, the size of the monetary policy shock for that month is set to 0. Figure [1] plots the monthly monetary policy shock estimates. A negative (positive) value of the HFI shock represents that the identified monetary policy shock is expansionary (contractionary) during that month. For example, the FOMC announced its plan to reduce the FFR target by 50 basis points to 4.5% after the meeting on April 18, 2001. This action surprised the market as the size of the reduction was larger than the market had anticipated. It led to a decrease in interest rates on all assets considered within the 30-minute window of the FOMC announcement.

2.2 Housing price index and housing supply elasticity

The housing price index used in this study is the Freddie Mac House Price Index (FMHPI). This seasonally adjusted series covers most MSAs since January 1975 at monthly frequency. The FMHPI
is constructed using a weighted repeat transactions methodology and is estimated with data including transactions on single-family detached and townhome properties serving as the collateral on loans purchased by Freddie Mac or Fannie Mae. This method is standard in housing research and is based on a dataset commonly used to construct other housing price indices such as the FHFA.

Data on MSA-level housing supply elasticities are based on estimates from Saiz (2010). This topology-based estimate is constructed based on physical and regulatory constraints in long-run housing construction in MSAs. It has been broadly used in other studies, such as Mian and Sufi (2009, 2011) and Mian et al. (2013), to identify the cross-sectional housing price heterogeneity.

To understand the cross-sectional identification, consider an aggregate shock that drives local housing demand uniformly. The slope of the local housing supply curve determines the degree to which housing prices rise in an area. Areas with inelastic housing supply experience larger changes in housing prices but smaller changes in quantities. Specifically, after an expansionary shock, the increase in the MSA-level housing price should be inversely proportional to the MSA-specific housing supply elasticity.

2.3 Estimation with Jordà local projection

My analysis in this section provides an estimate of how an MSA’s housing price, $y_{i,t+h}$, at horizon, $h$ behaves in response to a monetary policy shock at time $t$ given its housing supply elasticity, $\epsilon_i^{\text{supply}}$. I use a panel local projection method in the spirit of Jordà (2005). The original local projection method, a direct forecast method, provides robust nonparametric estimates to IRFs, $\delta y_{t+h}/\delta S_t$, by running a series of separate regressions for each horizon, $h$. This method is suitable if externally identified shocks are available and we are worried about misspecification issues in iterated methods such as structural VAR models. This method has been widely adopted in the recent literature on estimating the impacts of monetary policy shocks (see Jeenas, 2018) or fiscal policy shocks (see Ramey and Zubairy, 2018), thanks to the development in externally identified shocks.

To estimate the impulse response of the housing price growth in MSA $i$ $h$-periods after a monetary policy shock, I regress the cumulative growth of housing prices, $\Delta_h y_{i,t+h} = \log y_{i,t+h} - \log y_{i,t-1}$, on the interaction term of a monetary policy shock $S_t$ and MSA $i$’s housing supply
elasticity $\epsilon_i^{\text{supply}}$, alongside a number of control variables. Specifically, I estimate the following regression separately for $h = 0, 1, \ldots, H$.

$$\Delta_h y_{i,t+h} = \alpha_h + \beta_h S_t \times \epsilon_i^{\text{supply}} + \sum_{j=1}^{12} c_{h,j} \Delta y_{i,t-j} + \xi_i + \nu_t + u_{it} \quad (1)$$

I include 12 lags of changes in (log) house prices ($\Delta y_{i,t-j}$) as control variables and Location fixed effects ($\xi_i$) to capture any MSA-specific trends that might be correlated with the housing supply elasticity. I include time fixed effects ($\nu_t$) to capture the housing price growth driven by other concurrent aggregate shocks. Under this specification, the coefficients, $\beta_h$, are linear approximations of $\delta y_{i,t+h}/\delta S_t \delta \epsilon_i^{\text{supply}}$. The estimates measure local equilibrium effects of monetary policy shocks on housing prices due to the inclusion of fixed effects.

The key coefficient of interest, $\beta_h$, summarizes the dynamic difference in local house price responses to monetary policy shocks. Intuitively, given an expansionary monetary policy shock, MSAs with more elastic housing supply experience a larger increase in housing supply, and therefore a smaller increase in housing prices. As a result, one should expect negative $\beta_h$s after an expansionary monetary policy shock.

Figure 2 presents estimated $\beta_h$s up to three years. It represents the differential impulse response in the cumulative house price growth between two MSAs that differ in housing supply elasticity by one unit, for example, San Francisco and Philadelphia or Houston and Oklahoma City. The difference is the result of an expansionary monetary policy shock that effectively lowers the rate on the 10-year treasury-bill by 100 basis points. Standard errors are clustered at MSA level, allowing for arbitrary autocorrelation of the house price growth within each MSA.

These estimates of the $\beta_h$s provide only a comparative measurement of the house price response, but it is feasible to make a conservative inference on the overall response for each MSA. Assume that house prices in MSAs with housing supply elasticity above the 95% quantile ($\epsilon_i^{\text{supply}} \geq 5$) do not respond to monetary policy shocks. Now I can estimate the MSA-specific house price growth impulse response to a contractionary shock by simply calculating $(\epsilon_i^{\text{supply}} - 5) \times \beta_h$ for MSAs that are below the 95% threshold. Table 2 summarizes the estimates of the two-year housing price impulse
Table 2: MSA-level housing price responses to monetary policy shocks

<table>
<thead>
<tr>
<th>Percentile</th>
<th>$\epsilon_i^{\text{supply}}$</th>
<th>MSA</th>
<th>2-year HPI</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.7</td>
<td>San Diego, CA / New York, NY</td>
<td>11.91</td>
</tr>
<tr>
<td>10</td>
<td>1.2</td>
<td>Pittsburgh, PA / Baltimore, MD</td>
<td>10.53</td>
</tr>
<tr>
<td>25</td>
<td>1.6</td>
<td>Washington, DC / Philadelphia, PA-NJ</td>
<td>9.42</td>
</tr>
<tr>
<td>50</td>
<td>2.3</td>
<td>Houston, TX / Ann Arbor, MI</td>
<td>7.48</td>
</tr>
<tr>
<td>75</td>
<td>3.3</td>
<td>Kansas City, KS / Oklahoma City, OK</td>
<td>4.71</td>
</tr>
<tr>
<td>90</td>
<td>4.4</td>
<td>Iowa City, IA / Mansfield, OH</td>
<td>1.66</td>
</tr>
</tbody>
</table>

Notes: This table reports the MSA-level housing price response to an expansionary monetary policy shock at the two-year horizon. The shock effectively lowers the rate on the 10-year treasury bill by 100 basis points. Column 1 reports the percentile of the MSA’s housing supply elasticity. These estimates are calculated based on the Jordà local projection method.

2.4 Estimation with a proxy-SVAR model

In this section, I present a panel structural VAR model to estimate the structural IRFs of housing price growth to monetary policy shocks. This alternative method not only serves as a robustness check, but also links the empirical IRFs to their model counterparts introduced in Section 3. Unlike the local projection method, where the IRF for each horizon is estimated separately, the SVAR method provides statistical inference on the correlation over IRFs from different horizons. Although I no longer impose the same cross-sectional identification restriction, I show that estimated IRFs are broadly in line with an inverse relationship between the housing supply elasticity and the housing price response.

I identify the effect of structural monetary policy shocks in the VAR using external instruments measured from high-frequency financial market data. The estimation approach builds on methods pioneered by Stock and Watson (2012) and Mertens and Ravn (2013). Stock and Watson (2018) provides a detailed analysis of the identification and estimation of proxy-SVAR models and compares them to the local projection method. My VAR specification closely follows the work of Gertler and Karadi (2015) by combining both macro and financial variables at the aggregate level. I then modify the SVAR model to include local housing prices. The modified specification could easily
accommodate housing prices from all MSAs in one framework. For brevity, I present results for housing quartiles only.

Let \( y_t \) denote an \((n \times 1)\) vector of stationary macro variables including the Fed Funds rate (FFR), output growth, inflation, and excess bond premium. Let \( p_t \) denote a \((4 \times 1)\) vector whose \( j \)th element is the monthly growth rate of housing price quartile \( j \), and \( \bar{p}_t \) be the average price change over four quartiles. Let \( u_t \) be a \(((n + 4) \times 1)\) vector of structural shocks. I estimate the following VAR system with the sample from January 1990 to December 2007 with 12 lags \((L = 12)\):

\[
\begin{bmatrix}
y_t \\
p_t
\end{bmatrix} = c + \sum_{l=1}^{L} \begin{bmatrix}
\alpha_l & \beta_l \\
\gamma_l & \delta_l
\end{bmatrix} \begin{bmatrix}
y_{t-l} \\
p_{t-l}
\end{bmatrix} + Au_t. \tag{2}
\]

To estimate the effects of monetary policy shocks, one need only identify the first column of the structural matrix \( A \). I identify \( A_{i,1} \) by running the following 2SLS regression using HFI shocks \((z_t)\) discussed in Section 2.1 as external instruments for \( FFR_t \):

\[
x_{i,t} = A_{i,1} FFR_t + \phi_i(L)Y_t + v_{i,t}
\]

where \( x_{i,t} \) is the \( i \)th element in \( Y_t = (y_t, p_t)' \).

To construct the orthogonalized IRFs, I first compute the nonorthogonalized IRFs at horizon \( s \) from the reduced-form VAR \( \Psi_s = \partial Y_{t+s}/\partial \epsilon^t \). The orthogonalized IRFs of a monetary policy shock are simply \( \Psi_s A_{.,1} \). I construct the covariance matrix of the structural IRFs on four quartiles of housing prices over 36 horizons using a parametric bootstrap method. Details of estimating the IRFs and its covariance matrix are presented in Appendix A.

Estimated IRFs for each housing price quartile are reported in Figure 3. Note that in the estimation with the proxy-SVAR model, I do not impose the restriction of an inverse linear relationship between local house price growth and housing supply elasticity. A disadvantage of this method is that the IRFs are less accurately estimated with larger confidence bands. The results line up roughly with the prediction that housing prices in more inelastic quartiles increase more after an expansionary monetary policy shock. The two-year IRF estimate is about three-quarters as large
as the estimate from the Jordà local projection.

3 A housing model with information friction

This section generalizes an asset pricing model of housing from Glaeser and Nathanson (2017). The model is modified to allow for endogenous housing supply, which is a key aspect of the housing market. Housing prices exhibit different dynamics across cities after an aggregate demand shock due to diverse housing supply conditions. Households within each city are heterogeneous in their idiosyncratic housing preferences. They have imperfect information about the market fundamentals, but they form rational expectations. Details of the model setup are presented in Section 3.1. I discuss the solution method in Section 3.2 and derive housing price IRFs to monetary policy shocks in the model. I extend the model to account for search frictions in the housing market in Section 3.3.

3.1 Housing demand and housing supply

In a large economy with one central bank and several cities, each city differs in its housing supply elasticity due to distinct geographical and regulatory constraints. Consider a city with a housing supply elasticity $\chi$. An individual household $i$ in this city receives a flow of utility improvement $D_{i,t}$ if she decides to buy one unit of house. I adopt the notion in Glaeser and Nathanson (2017) that this overall utility combines an idiosyncratic element, $a_{i,t}$, with a city-specific component, $D_t$:

$$D_{i,t} = D_t + a_{i,t}$$

These elements include returns from the local labor market and the utility yielded from living in the current city. The idiosyncratic component $a_{i,t}$ is drawn independently at every period for each individual from a normal distribution $N \sim (0, \sigma_a^2)$. The city-specific component, $D_t$, is subject to both city-specific and aggregate shocks such as monetary policy shocks. I will specify it later when I discuss the dynamics of the fundamental states. Overall, $D_{i,t}$ captures an individual’s marginal propensity to live in a specific city.

\[^2\]We assume $a_{i,t} \sim i.i.d.$ for simplicity, but our result is robust to modifying the process of $a_{i,t} \sim AR(1)$ to allow for a persistent but idiosyncratic process across each individual.
Given an individual’s utility flow of $D_{i,t}$, and a price of $P_{i,t}$ for a preferred house type, her quantity demanded for housing, $H_{i,t}$, satisfies

$$p_{i,t} = D_{i,t} - \frac{h_{i,t}}{\chi_D} + \beta \mathbb{E}_t P_{i,t+1},$$

(3)

where $p_{i,t} = \log P_{i,t}$, $h_{i,t} = \log H_{i,t}$, and $\chi_D$ is the individual housing demand elasticity. Equation (3) characterizes the individual housing demand in my model. To interpret it from an asset pricing perspective, $p_{i,t}$ is the individual’s willingness to pay for the house, and it equals the discounted value of owning the house. $D_{i,t} - h_{i,t}/\chi_D$ is simply the marginal utility from owning $H_{i,t}$ units of housing and it diminishes as the amount of housing increases. This housing demand equation can also be motivated from the perspective of a utility-maximizing household in a typical DSGE model with housing: $D_{i,t}$ approximates the household’s income and propensity towards housing.

In the baseline model, I approximate the housing supply with a log-linear relationship between the housing price and the housing quantity, subject to housing supply shocks that are orthogonal to housing demand shocks. Given price $P_{i,t}$, the available quantity of this type of housing in this city with housing supply elasticity $\chi_S$ is

$$h_{i,t} = \chi_S p_{i,t} + s_t,$$

(4)

where $s_t$ denotes city-level housing supply shocks, which follow an $AR(1)$ process with persistence $\mu_s$ and noise $\sigma_s$. Equation (4) captures the important feature that the housing quantity is more sensitive to housing price fluctuations in cities with a more elastic housing supply. I use this linear housing supply equation since the focus of this paper is to study the dynamics of housing prices under monetary policy shocks. I assume that the short-run fluctuation in interest rate does not transmit directly to the supply of housing, but only through housing prices.\footnote{For models that focus on the dynamics of housing volume and its relationship to prices (see Nathanson and Zwick, 2018; DeFusco et al., 2017 and others).}

Let $p^*_i$ denote the housing price that clears the market of an individual house type $i$. I define the aggregate housing price index for this city as the average housing price $p_t = \int p^*_i d_i$. Therefore,
the housing price index in each city satisfies the following law of motion.

\[ p_t = \frac{D_t}{1 + \chi^S/\chi^D} + \frac{\beta}{1 + \chi^S/\chi^D} \bar{E}_t p_{t+1} - \frac{s_t}{\chi^D}, \]

(5)

where \( \bar{E}_t p_{t+1} = \int E_{it} p_{i,t+1} di \), which is the average expectation of future housing prices across all households. Given a demand shock, the housing price response depends on the dynamics of market fundamentals and the evolution of household expectation formation. I specifically model the role of information frictions in driving households’ expectation and subsequently their effects on short-run housing price dynamics.

Local housing market fundamentals are driven by both local demand factors and aggregate demand shocks. I model the demand for housing in city \( m \) as

\[ D_{mt} = T_{mt} + \alpha Q_t, \]

(6)

where \( T_{mt} \) represents the local demand factor arising from local amenities. This factor captures the utility flow of an average household in the overall benefit of living in this city. \( Q_t \) represents the aggregate demand factor that affects all cities uniformly. Since the focus is on the monetary policy, \( Q_t \) captures the effect of monetary policy on housing demand. \( \alpha \) is a scaling factor that weighs the relative importance of the monetary policy to local demand. Demand factors \( T_{mt} \) and \( Q_t \) evolve over time as

\[ T_{mt} = T_{m,t-1} + g_{mt} + \epsilon^d_{mt}, \quad \epsilon^d_{mt} \sim i.i.d. N(0, \sigma^2_d) \]

(7)

\[ g_{mt} = \rho g_{m,t-1} + \epsilon^g_{mt}, \quad \epsilon^g_{mt} \sim i.i.d. N(0, \sigma^2_g) \]

(8)

\[ Q_t = \theta Q_{t-1} + \epsilon^{MP}_t, \quad \epsilon^{MP}_t \sim i.i.d. N(0, \sigma^2_r) \]

(9)

where \( \epsilon^d_t \) and \( \epsilon^g_t \) are shocks to the level and growth of local demand. A shock \( \epsilon^d_t \) has a permanent impact on the utility of living in this city. For example, a city’s investment in a good school district

\[ \text{Note that I have extended the housing price model in Glaeser and Nathanson (2017) to allow for variations in housing supply conditions. This model coincides with their setup in discrete time if it is restricted to the extreme case of inelastic housing supply, } \chi^S = 0. \]
is a positive shock to local housing demand, while closing down large manufacturing plants serves as a negative housing demand shock. A shock to the growth of demand \( \epsilon_t^g \) has persistent impacts on housing. This could come from news about future investments in the city. For example, housing demand increases as people expect that a large company is building new headquarters in this city. \( \epsilon_t^{MP} \) denotes monetary policy shocks, which has a persistence of \( \theta \).

Individual household demand for housing is heterogeneous due to idiosyncratic preferences. I assume individual housing demand is normally distributed around the city aggregate demand \( D_{mt} \):

\[
D_{it} = D_{mt} + a_{it}. \tag{10}
\]

Each household knows its own demand for housing, but does not fully observe the demand at the city level. Instead, the household observes a common signal \( D_{mt}^s \) about the housing demand:

\[
D_{mt}^s = D_{mt} + a_{mt}, \quad a_{mt} \sim i.i.d. N(0, \sigma_a^2). \tag{11}
\]

The household also observes the aggregate housing price history in the local market up to the current period, \( \{p_{t-j}\}_{j=0}^{\infty} \), and a sequence of common signals of monetary policy stances \( \{r_{t-j}\}_{j=0}^{\infty} \):

\[
r_t = Q_t + q_t, \quad q_t \sim i.i.d. N(0, \sigma_q^2) \tag{12}
\]

To summarize, an individual household’s information set at period \( t \) is \( \Omega_{it} = \{y_{it}, y_{i,t-1}, \ldots, y_{i1}\} \) where \( y_{it} = (p_{mt}, D_{mt}^s, D_{it}, r_{t})' \).

### 3.2 Solution to the model

In this section, I propose a solution method that computes the housing price dynamics given the fundamental demand. This problem is unconventional since the housing price is an equilibrium object that depends on the households’ decisions on housing demand. Meanwhile, households make decisions using the current price as a signal about market conditions. I solve this model by formulating it in a state-space framework in which the state matrix is computed through a fixed-point
iteration.

I start with the equilibrium housing price. Households’ information sets differ only in their individual demands, \(D_{it}\), whose idiosyncratic parts, \(a_{it}\), are independently normally distributed. When the equilibrium housing price is linear in housing demand, \(\int \mathbb{E}_t p_{i,t+1} di = \mathbb{E}_t (p_{m,t+1} | \Omega_{mt})\), where \(\Omega_{mt} = \{y_{mt}, y_{m,t-1}, \ldots, y_{m1}\} \) and \(y_{mt} = (p_{mt}, D^s_{mt}, r_t)'\). According to Equation (5), the aggregate housing price in city \(m\) with housing supply elasticity \(\chi^S_m\) follows

\[
p_{mt} = s_{mt} + D_{mt} + \beta_m \mathbb{E}_t (p_{m,t+1} | \Omega_{mt})
\]

\[
= s_{mt} + T_{mt} + \alpha_m Q_t
\]

\[
+ \mathbb{E}_t \{[\beta_m (s_{m,t+1} + T_{m,t+1} + \alpha_m Q_{t+1}) + \beta_m^2 (s_{m,t+2} + T_{m,t+2} + \alpha_m Q_{t+2}) + \cdots]| \Omega_{mt}\}
\]

\[
= s_{mt} + T_{mt} + \alpha_m Q_t + \frac{\alpha_m \beta_m \theta}{1 - \beta_m} Q_t + \frac{\beta_m}{(1 - \beta_m)(1 - \beta_m \rho)} \tilde{g}_{m,t|t},
\]  

(13)

where \(s_{mt} = -s_{mt} / \chi^S_m\), \(D_{mt} = \frac{1}{1 + \chi^S_m / \chi^D} D_{mt}\), \(T_{mt} = \frac{1}{1 + \chi^S_m / \chi^T} T_{mt}\), \(\tilde{g}_{mt} = \frac{1}{1 + \chi^S_m / \chi^g} g_{mt}\), \(\alpha_m = \frac{1}{1 + \chi^S_m / \chi^D} \alpha\) and \(\beta_m = \frac{1}{1 + \chi^S_m / \chi^T} \beta\). \(X_{m,t|t} = \mathbb{E}_t (X_t | \Omega_{mt})\), \(X = T, Q, g\), denotes optimal inference given the information set \(\Omega_{mt}\).

I assume no short-run interaction between local housing markets. The solution method applies uniformly to each local market. For simplicity, I drop the subscript \(m\) in the following analysis. The city-specific housing price behaves as if there is a representative agent in the housing market with information set \(\Omega_t\). But the equilibrium price depends not only on the fundamental demand at period \(t\), but also on the household’s best inference about them. At this stage, the key step is to solve for the dynamics of \(Q_{t|t}, T_{t|t},\) and \(g_{t|t}\).

I solve for the dynamics of \(\Phi_{t|t} = \{T_{t|t}, g_{t|t}, Q_{t|t}\}\) by finding a fixed point in the Kalman filter recursion. Specifically, I develop an algorithm to find the coefficient matrix \([c_p, c_q, c_d]\) in

\[
\Phi_{t|t} = \Phi_{t|t-1} + c_p (p_t - p_{t|-1}) + c_q (r_t - Q^s_{t|-1}) + c_d (D^s_t - D^s_{t|-1}).
\]  

(14)

Details of the solution method are presented in Appendix B.

Once I find the fixed point for the coefficient matrix, I establish a law of motion for the state
variables and its inferences \( (\Phi_t', \Phi_{t|t}') \) as

\[
\begin{bmatrix}
\Phi_t \\
\Phi_{t|t}
\end{bmatrix} =
\begin{bmatrix}
F & 0 \\
B_1 & B_2
\end{bmatrix}
\begin{bmatrix}
\Phi_{t-1} \\
\Phi_{t-1|t-1}
\end{bmatrix} +
\begin{bmatrix}
\epsilon_t \\
\epsilon^X_t
\end{bmatrix},
\]

where

\[ F = \begin{bmatrix} 1 & 1 & 0 \\ 0 & \rho & 0 \\ 0 & 0 & \theta \end{bmatrix} \]

\[ B_1 = \begin{bmatrix} c_p & c_q & c_d \end{bmatrix} H_1 F \]

\[ B_2 = F - \begin{bmatrix} c_p & c_q & c_d \end{bmatrix} H_1 F \]

\[ \epsilon^X_t = \begin{bmatrix} c_p & c_q & c_d \end{bmatrix} H_2 v_t + \begin{bmatrix} c_p & c_q & c_d \end{bmatrix} H_1 \epsilon_t. \]

\( H_1 \) and \( H_2 \) are specified in Appendix B.

To find the housing price IRFs to monetary policy shocks, I rewrite Equation (13) as a matrix:

\[
p_t = \begin{bmatrix} 1 & 0 & x \end{bmatrix} \begin{bmatrix}
\frac{\beta}{1 - \beta} & \frac{\beta}{(1 - \beta)(1 - \beta \rho)} & \frac{\alpha \beta \theta}{1 - \beta \theta} \\
\frac{1}{1 - \beta} & \frac{1}{(1 - \beta)(1 - \beta \rho)} & \frac{1}{1 - \beta \theta}
\end{bmatrix}
\begin{bmatrix}
\Phi_t \\
\Phi_{t|t}
\end{bmatrix} + s_t. \tag{16}
\]

Equations (15) and (16) imply the structural model’s IRF is

\[
\frac{\partial p_{t+h}}{\partial e^M_{t}} = \begin{bmatrix}
1 & 0 & x \end{bmatrix} \begin{bmatrix}
\frac{\beta}{1 - \beta} & \frac{\beta}{(1 - \beta)(1 - \beta \rho)} & \frac{\alpha \beta \theta}{1 - \beta \theta} \\
\frac{1}{1 - \beta} & \frac{1}{(1 - \beta)(1 - \beta \rho)} & \frac{1}{1 - \beta \theta}
\end{bmatrix}
\begin{bmatrix}
F & 0 \\
B_1 & B_2
\end{bmatrix}^h
\begin{bmatrix}
\epsilon_3 \\
\epsilon^X_t
\end{bmatrix} + \begin{bmatrix}
\epsilon_3 \\
\epsilon^X_t
\end{bmatrix} H_3 e_3, \tag{17}
\]

where \( e_3 = (0, 0, 1)' \).

### 3.3 Search frictions

Suppose some households take some time to search before purchasing a house so that their demand for housing after a shock to \( D_t \) is realized with some delay. I describe the cross-sectional distribution
of search time with a Weibull density as in Hamilton (2008). Let $\omega_j$ denote the fraction of households who searched $j$ months for houses predicted by the Weibull density with a scale parameter $\lambda$ and a shape parameter $\kappa$. In this case, the equilibrium price is

$$p_t = s_t + \omega_0 \left( T_t + \alpha Q_t + \frac{\beta}{1 - \beta} T_{t|t-1} + \frac{\alpha \beta \theta}{1 - \beta \theta} Q_{t|t-1} + \frac{\beta}{(1 - \beta)(1 - \beta \rho)} g_{t|t-1} \right) + \sum_{j=1}^{\infty} \omega_j \left( \frac{1}{1 - \beta} T_{t|t-1} + \frac{\alpha}{1 - \beta \theta} Q_{t|t-1} + \frac{\beta}{(1 - \beta)(1 - \beta \rho)} g_{t|t-1} \right).$$

I can solve the model using the same technique except that

$$p_t - p_{t|t-1} = s_t + \omega_0 \left[ D_t - D_{t|t-1} + \frac{\beta}{1 - \beta} (T_{t|t} - T_{t|t-1}) + \frac{\alpha \beta \theta}{1 - \beta \theta} (Q_{t|t} - Q_{t|t-1}) \right] + \frac{\beta}{(1 - \beta)(1 - \beta \rho)} (g_{t|t} - g_{t|t-1})].$$

This implies that $p_t$ carries less information about $D_t$ when $\omega_0 < 1$ because only a fraction of households receives signals about the current state.

Once I solve for $(a_0, a_2, c_1', c_2')$ in a similar fashion, I can compute the IRFs as

$$\frac{\partial p_{t+h}}{\partial e_t^{MP}} = \omega_0 \phi_h + \sum_{j=1}^{h} \omega_j \psi_{h-j}$$

$$\phi_h = \begin{bmatrix} 1 & 0 & \frac{\beta}{1 - \beta} & \frac{\alpha \beta \theta}{1 - \beta \theta} \end{bmatrix} \begin{bmatrix} F & 0 & e_4 \\ B_1 & B_2 \end{bmatrix}^{\text{h}} H_1 e_4$$

$$\psi_h = \begin{bmatrix} 0 & 0 & \frac{\beta}{1 - \beta} & \frac{\alpha \beta \theta}{1 - \beta \theta} \end{bmatrix} \begin{bmatrix} F & 0 & e_3 \\ B_1 & B_2 \end{bmatrix}^{\text{h}} H_1 e_3.$$

4 Structural estimation

To understand the degree of information rigidity in the transmission of monetary policy shocks to housing markets, I estimate the structural parameters in the model from Section 3. The empirical study in Section 2.4 has provided estimates on the impulse responses of housing prices at different
Table 3: Calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_D$, $\sigma_q$</td>
<td>1</td>
<td>Normalization of demand and monetary policy shocks</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.96^{1/12}</td>
<td>Discount factor (monthly frequency)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.5^{1/6}</td>
<td>Persistence of housing demand growth (Glaeser and Nathanson, 2017)</td>
</tr>
<tr>
<td>$\sigma_g$</td>
<td>0.1</td>
<td>Standard deviations of shocks to housing demand growth</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.99</td>
<td>Persistence of monetary policy shock</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.08</td>
<td>Targeting a semi-elasticity of housing price to interest rate of 6%</td>
</tr>
<tr>
<td>$(\lambda, \kappa)$</td>
<td>(14.7/4, 0.972)</td>
<td>Weibull density scale and shape parameters, Hamilton (2008)</td>
</tr>
</tbody>
</table>

quartiles to a monetary policy shock. A simple way to estimate the model’s structural parameters is to match its predicted moments to their empirical counterparts. In this section, I estimate the model’s signal-to-noise ratio, $\Lambda = (\sigma_a, \sigma_s, \sigma_r)'$, using a minimum chi-square estimator.

Given the complexity of the model, it is infeasible to estimate all structural parameters simultaneously. Some model parameters have been well studied in the literature. In my estimation, I calibrate a subset of parameters using existing findings to focus on the ones that directly measure the degree of information frictions. Table 3 presents the calibrated parameters.

Let $\hat{\pi} = (\hat{\pi}_1', \hat{\pi}_2', \hat{\pi}_3', \hat{\pi}_4')$ denote the empirical estimates of housing price IRFs in every quartile, where $\hat{\pi}_i = (\Phi_{i,0}, \Phi_{i,1}, \ldots, \Phi_{i,36})$ is the vector of IRF estimates plotted in Figure 3 from Section 2.4. Let $\pi(\Lambda) = (\pi_1'(\Lambda), \pi_2'(\Lambda), \pi_3'(\Lambda), \pi_4'(\Lambda))'$ denote the model-implied IRFs of housing prices to monetary policy shocks as calculated in Equation (17), with $\pi_i(\Lambda)$ being the IRFs using the $i$th quartile housing supply elasticity. The minimum chi-square estimator for $\Lambda = (\sigma_a, \sigma_s, \sigma_r)'$ satisfies

$$\Lambda = \arg \min_\Lambda T[\hat{\pi} - \pi(\Lambda)]'\hat{V}^{-1}[\hat{\pi} - \pi(\Lambda)],$$

where $\hat{V}$, the weighting matrix, is the bootstrapped covariance matrix estimated in Section 2.4. I estimate the parameters through a numerical optimization. The estimates are reported in Table 4. Since I have normalized the size of local demand shocks, $\sigma_D$, and monetary policy shocks $\sigma_q$ to one, the sizes of the noise in household signals is interpreted as noise-to-signal ratios. One period in the model corresponds to one month. At a monthly frequency, shocks are small and infrequent while signals reveal little new information about market fundamentals. The size of housing supply
shocks, $\sigma_s$, measures the quality of local housing prices as a signal for housing demand. Its size of 15.7 suggests that it takes about five quarters for households to infer a monetary policy shock through housing prices alone. The standard error of noise in demand signals, $\sigma_a$, measures the relative change in households’ perception of housing market fundamentals to actual changes. Its estimate of 24.9 suggests that it takes about two years for households to infer a monetary policy shock through their housing demand signals. The standard error of noise in monetary policy signals, $\sigma_q$, measures the precision of households’ perceptions about monetary policy. The noise in this signal is huge and I have capped its size at 100 in the numerical optimization. This result suggests that households pay little attention to changes in monetary policy stances and do not associate them with the local housing market. After an expansionary monetary policy shock, households gradually learn about the impact on local housing markets through house price changes and local news about housing demand. Households adjust their housing demand based on this information. In equilibrium, the local house price rises as the housing demand adjusts.

### Table 4: Estimated structural parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\sigma_s$</th>
<th>$\sigma_a$</th>
<th>$\sigma_q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimates</td>
<td>15.7</td>
<td>24.9</td>
<td>100</td>
</tr>
<tr>
<td>Standard errors</td>
<td>1.12</td>
<td>2.74</td>
<td>—</td>
</tr>
</tbody>
</table>

I compute the standard errors of estimated structural parameters by estimating the covariance matrix for $\hat{\Lambda}$ as $\text{cov}(\hat{\Lambda}) = T^{-1}(\hat{\Gamma}'\hat{\Sigma}^{-1}\hat{\Gamma})^{-1}$, where $\hat{\Gamma} = \partial \pi(\theta)/\partial \Lambda|_{\theta = \hat{\theta}}$ (see Hamilton and Wu, 2012). The estimated standard errors are reported in the bottom row of Table 4.

Figure 4 presents the model-predicted housing price IRFs to monetary policy shocks using the estimated structural parameters. From top to bottom, each curve represents the IRFs from cities within each quartile of housing supply elasticity, with a larger response for cities with a less elastic housing supply. Model-predicted IRFs align well with their empirical counterparts in Figure 3.

### 5 Implications to the 2000s’ housing boom

In this section, I study the 2000s’ housing boom by applying my estimates on the housing price response to monetary policy shocks. The federal reserve cut the FFR aggressively starting in January...
uary 2001 in anticipation of an upcoming recession. The FFR remained low for more than three years until the Fed started raising rates again in July 2004. During this period, the FFR deviated significantly from the Taylor rule, which is proposed by Taylor (1993) to describe the path of interest rates in the U.S. and commonly adopted in monetary macro models. In particular, Taylor (2007) claimed the deviation in monetary policy from the Taylor rule contributed to the 2000s’ housing boom. Given the FFR deviations, I evaluate its contribution to local housing prices using my estimates.

I first estimate the following regression which resembles the Taylor rule for the period January 1990–December 2008.

\[
\Delta R_t = c + \alpha \Delta R_{t-1} + \sum_{l=0}^{4} \beta_l \Delta \log(y_{t-l}) + \sum_{l=0}^{4} \gamma_l \Delta \log(p_{t-l}) + \sum_{l=0}^{4} \delta_l \Delta \log(HPI_{t-l}) + \sum_{l=0}^{4} \theta_l \Delta \log(UR_{t-l}) + \epsilon_t
\]

(19)

where \( R_t \) is the FFR, \( y_t \) is industrial production, \( p_t \) is the consumer price index, \( HPI_t \) is the housing price index, and \( UR_t \) is the unemployment rate. The original Taylor rule only includes output \( y_t \) and inflation \( p_t \). I have included more variables to reflect the fact that the Fed monitors a large number of indicators when making FFR decisions. I accumulate estimated residuals, \( \hat{\epsilon}_t \), from January 2001 to December 2007 to estimate the FFR’s deviation from the Taylor rule. Figure 5 shows the path of the effective FFR and the prediction from the Taylor rule. The difference between two paths is the deviation.

This deviation is a result of a sequence of monetary policy shocks estimated as \( \hat{\epsilon}_t \). Using the estimated structural model from Section 4, I simulate paths of housing price responses given any monetary policy shock in each city using its housing supply elasticity. For each shock, \( \hat{\epsilon}_t \), I compute the housing price IRF in city \( i \) for each horizon \( s \) as \( \Phi_{i,t+s} \). For each period, I accumulate impacts from all previous shocks to obtain the cumulative effect due to this Taylor deviation. Figure 6 plots the actual price growth and the price growths attributed to the Taylor deviation in two cities, San Francisco and Oklahoma City. In San Francisco, this deviation explains the majority of the housing price increase up to mid-2003. However, housing prices increased by about 60% at the peak, but
the monetary policy deviation explains only about a quarter of the boom. In Oklahoma City, the Taylor deviation consistently explains about a quarter of the housing price increase in spite of a more moderate housing price increase.

6 Conclusions

This paper studies the effect of monetary policy shocks on local housing prices. Empirically, I provide estimates on differential responses of U.S. local housing prices to aggregate monetary policy shocks. The results are robust using different estimation strategies. I propose a new housing price model with information friction and provide a simple method to estimate its structural parameters. The estimated result provides implications on the role of monetary policy in the 2000s’ housing boom.
References


This figure plots monthly monetary policy shocks from January 1991 to June 2008. Shocks are estimated using the high-frequency identification method. One unit of the shock is normalized to effectively raise the rate on the 10-year treasury-bill by 100 basis points. See Section 2.1 for details.
The solid dark blue line plots the differential impulse response of cumulative housing price growth between two MSAs that differ in housing supply elasticity by one unit, after an expansionary monetary policy shock that effectively lowers the rate on the 10-year treasury bill by 100 basis points. The dotted red lines outline the 95% confidence interval. See Section 2.3 for a detailed discussion.
These curves show impulse response functions of the cumulative housing price growth to an expansionary monetary policy shock that lowers the federal funds rate by 1%, at the first (red), second (pink), third (green), and fourth (blue) housing supply elasticity quartile. Estimates are computed using the proxy-SVAR method. See Section 2.4 for a detailed discussion.
These curves show impulse response functions of the cumulative housing price growth to an expansionary monetary policy shock that lowers the federal funds rate by 1%, at the first (red), second (pink), third (green), and fourth (blue) housing supply elasticity quartile. Estimates are computed using the estimated structural model in Section 4.
Figure 5: Predicted path of interest rates from the Taylor rule

The dark blue line shows the effective federal funds rate from January 2001 to December 2008. The red line shows the predicted path of interest rates from an estimated Taylor rule in Section 5.
Housing price growth for the period January 2001–December 2008 is shown for San Francisco and Oklahoma City in the top and bottom panels, respectively. In each panel, the dark blue line shows the actual path of the housing price growth, the red line shows the path of the housing price growth implied from the Taylor deviation. See Section [5] for details.
Appendices

A Estimation with the proxy-SVAR model

This appendix presents details of the estimation of the proxy-SVAR model in Section 2.4. The VAR model in Equation (2) is equivalent to

\[
(I - \Phi(L)) \begin{bmatrix} y_t \\ p_t \end{bmatrix} = \epsilon_t = Au_t,
\]

where

\[
\Phi_l = \begin{bmatrix} \alpha_l & \beta_l/J, \ldots, \beta_l/J \\ \gamma_l & \delta_l/J, \ldots, \delta_l/J \end{bmatrix}, \quad J = 4.
\]

Based on this VAR specification, I first compute the nonorthogonalized IRFs, \(\Psi_s\), for each horizon, \(s\), from the reduced-form VAR in a standard way (see Hamilton 1994 for details). The orthogonalized IRFs of a monetary policy shock are simply \(\Phi_s = \Psi_s A_{i,1}\), where \(A_{i,1}\) is identified by running the 2SLS regression using HFI shocks \((z_t)\) discussed in Section 2.1 as external instruments for \(FFR_t\):

\[
x_{i,t} = A_{1,i} FFR_t + \phi_i(L)Y_t + v_{i,t},
\]

where \(x_{i,t}\) is the \(i\)th element in \(Y_t = (y_t, p_t)'\).

Next, I compute the covariance matrix of the structural IRFs, \(\Phi(p_{j,s})\) for \(j = 1, \ldots, 4\) and \(s = 0, 1, \ldots, 36\). This is done with 1,000 draws from a parametric bootstrap. For each draw, I generate a sample of size \(T\) equivalent to my original sample time length for \((\tilde{Y}_t, \tilde{z}_t)\) from the stationary VAR

\[
\begin{bmatrix} I - \hat{\Phi}(L) \\ 0 \end{bmatrix} \begin{bmatrix} Y_t \\ z_t \end{bmatrix} = \begin{bmatrix} \tilde{\epsilon}_t \\ \tilde{\epsilon}_t \end{bmatrix},
\]

where

\[
\begin{bmatrix} \tilde{\epsilon}_t \\ \tilde{\epsilon}_t \end{bmatrix} \sim \text{i.i.d. } N \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma_\epsilon & \Sigma_{\epsilon\epsilon} \\ \Sigma_{\epsilon\epsilon} & \Sigma_\epsilon \end{bmatrix} \right),
\]

31
where \( \hat{\Phi}(L) \) is estimated from the reduced-form VAR and the \( \Sigma_s \) are sample covariances for VAR residuals \( \hat{\epsilon}_t \) and instruments \( z_t \). For each draw \( n = 1, 2, \ldots, 1,000 \), I compute the structural IRFs, \( \Phi^{(n)}_{j,s} \), as illustrated above. The sample covariance of \( \Phi^{(n)}_{j,s} \) provides the bootstrapped covariance matrix of \( IRF(p_{j,s}) \).

**B Solution to the housing price model**

This appendix presents details in the solution to the housing price model in Section 3.2. State Equations (7)–(9) can be written in matrix form as

\[
\begin{bmatrix}
T_{m,t+1} \\
g_{m,t+1} \\
Q_{t+1}
\end{bmatrix}
= 
\begin{bmatrix}
1 & 1 & 0 \\
0 & \rho & 0 \\
0 & 0 & \theta
\end{bmatrix}
\begin{bmatrix}
T_{mt} \\
g_{mt} \\
Q_{t}
\end{bmatrix}
+ 
\begin{bmatrix}
\epsilon_{m,t+1}^d \\
\epsilon_{m,t+1}^g \\
\epsilon_{t+1}^{MP}
\end{bmatrix}.
\]

Let \( \Phi_{m,t} = (T_{m,t+1}, g_{m,t+1}, Q_{t+1})' \) denote state variables and \( F \) denote the state matrix:

\[
\Phi_{m,t+1} = F\Phi_{m,t} + \epsilon_{m,t+1}.
\]

From here, I drop the subscript \( m \) since the solution method applies uniformly to every city. I conjecture that households at period \( t \) have a prior of \( \Phi_{t|t-1} \) with variance \( \Sigma_{t|t-1} = E(\Phi_t - \Phi_{t|t-1})^2 \). Note that

\[
p_{t|t-1} = \frac{1}{1 - \beta} T_{t|t-1} + \frac{\alpha}{1 - \beta \theta} Q_{t|t-1} + \frac{\beta}{(1 - \beta)(1 - \beta \rho)} g_{t|t-1}
\]

\[
D^s_{t|t-1} = D_{t|t-1}
\]

\[
r_{t|t-1} = Q_{t|t-1}
\]

Let \( y_t = (p_t, r_t, D^s_t) \) be observed variables. The next step is to solve for the updating equation to find \( \Phi_{t|t} \).

\[
\Phi_{t|t} = \Phi_{t|t-1} + c_p(p_t - p_{t|t-1}) + c_q(r_t - Q_{t|t-1}) + c_d(D^s_t - D^s_{t|t-1})
\] (20)
The coefficient matrix \[
\begin{bmatrix}
c_{pt} & c_{qt} & c_{dt}
\end{bmatrix}
\] is an unknown \(4 \times 3\) matrix that satisfies
\[
\begin{bmatrix}
c_{pt} & c_{qt} & c_{dt}
\end{bmatrix} = \Sigma_{yy,t}^{-1} \Sigma_{y\Phi,t},
\]
(21)
where
\[
\Sigma_{yy,t} = \mathbb{E}[(y_t - y_{t-1})(y_t - y_{t-1})']
\]
\[
\Sigma_{y\Phi,t} = \mathbb{E}[(y_t - y_{t-1})(\Phi_t - \Phi_{t-1})']
\]
\[
\Phi_{t+1|t} = F\Phi_{t|t}
\]
\[
\Sigma_{t|t} = \Sigma_{t|t-1} - \Sigma_{y\Phi,t} \Sigma_{yy,t}^{-1} \Sigma_{y\Phi,t}
\]
\[
\Sigma_{t+1|t} = F\Sigma_{t|t} F' + Q
\]
To implement the recursion, note that
\[
\begin{align*}
p_t - p_{t|t-1} &= s_t + D_t - D_{t|t-1} + \frac{\beta}{1 - \beta} (T_{t|t} - T_{t|t-1}) + \frac{\alpha \beta \theta}{1 - \beta \rho} (Q_{t|t} - Q_{t|t-1}) \\
& \quad + \frac{\beta}{(1 - \beta)(1 - \beta \rho)} (g_{t|t} - g_{t|t-1}) \\
& = s_t + D_t - D_{t|t-1} + \left[ \frac{\beta}{1 - \beta} \frac{\beta}{(1 - \beta)(1 - \beta \rho)} \frac{\alpha \beta \theta}{1 - \beta \rho} \right] \begin{bmatrix} c_{pt} & c_{qt} & c_{dt} \end{bmatrix} \begin{bmatrix} p_t - p_{t|t-1} \\ r_t - Q_{t|t-1} \\ D_t^a - D_{t|t-1}^a \end{bmatrix}.
\end{align*}
\]
After rearranging,
\[
p_t - p_{t|t-1} = s_t^a + a_{qt} (D_t - D_{t|t-1}) + a_{dt} (D_t^a - D_{t|t-1}^a) + a_{qt} (r_t - Q_{t|t-1}),
\]
where

\[
\begin{align*}
    a_{0t} &= \frac{1}{1 - \left[ \frac{\beta}{1-\beta} \left( \frac{\beta}{1-\beta(1-\rho)} \right) \right] c_{pt}} \\
    a_{xt} &= a_{0t} \left( \left[ \frac{\beta}{1-\beta} \left( \frac{\beta}{1-\beta(1-\rho)} \right) \right] c_{xt} \right), \quad x \in \{d, q\} \\
    s^*_t &= a_{0t} s_t.
\end{align*}
\]

We can now see that

\[
\begin{align*}
    y_t - y_{t|t-1} &= H_{1t}(\Phi_t - \Phi_{t|t-1}) + H_{2t} \begin{pmatrix} s_t \\ q_t \\ a_t \end{pmatrix} \\
    \Sigma_{y_0, t} &= H_{1t}\Sigma_{\theta_{t|t-1}}H_{1t}' + H_{2t}R H_{2t}' \\
    \Sigma_{y, t} &= H_{1t}\Sigma_{\theta_{t|t-1}}.
\end{align*}
\]

where \(H_{1t} = \begin{bmatrix} a_{0t} + a_{dt} & 0 & a_{qt} + a(a_{0t} + a_{dt}) \\ 0 & 0 & 1 \end{bmatrix}, H_{2t} = \begin{bmatrix} a_{0t} & a_{qt} & a_{dt} \\ 0 & 1 & 0 \end{bmatrix}, \) and \(R = \begin{bmatrix} \sigma_s^2 & 0 & 0 \\ 0 & \sigma_q^2 & 0 \\ 0 & 0 & \sigma_a^2 \end{bmatrix}.\)

The solution to this model is a fixed point to Equation (21) in which the matrix \([c_{pt}, c_{qt}, c_{dt}]\) calculated from the right-hand side has the same values on the left-hand side. The fixed point can be found through convergence in \(\Sigma_{t+1|t}.\) I denote its value at the fixed point as \((a_0, a_q, a_d, c'_p, c'_q, c'_d).\)